VECTORS

(Estimated time to complete this section: 2 hours)

Many of the quantities in physics are vectors. ***<u>This makes proficiency in vectors extremely important</u>***.

Magnitude: Size or extent. The numerical value.

Direction: Alignment or orientation of any position with respect to any other position.

Scalars: A physical quantity described by a single number and units. A quantity described by **<u>magnitude only</u>**.

Examples: time, mass, and temperature

Vector: A physical quantity with **<u>both a magnitude and a direction</u>**. A directional quantity.

Examples: velocity, acceleration, force

Notation: $\vec{A}$ or $\vec{A}$ ⟶

<u>Length</u> of the arrow is <u>proportional to the vectors magnitude</u>.
<u>Direction</u> the arrow points <u>is the direction of the vector</u>.

Negative Vectors

Negative vectors have the same magnitude as their positive counterpart. They are just pointing in theopposite direction.

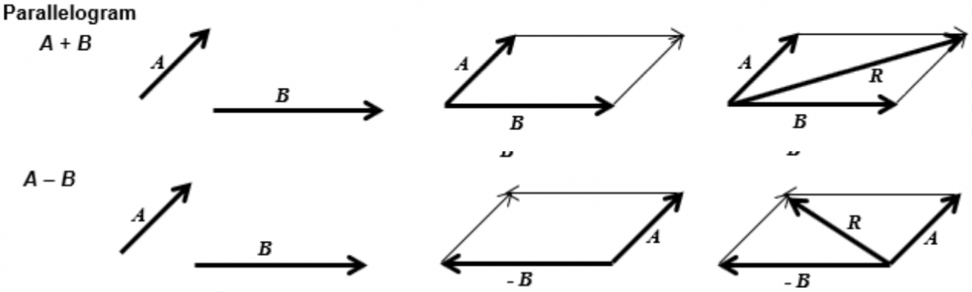
$\vec{A}$ ⟶ ◀ $-\vec{A}$

Vector Addition and subtraction

<u>This is very important</u>. In physics a negative number does not always mean a smaller number. Mathematically –2 is smaller than +2, but in physics these numbers have the same magnitude (size), theyjust point in different directions (180° apart).

There are two methods of adding vectors

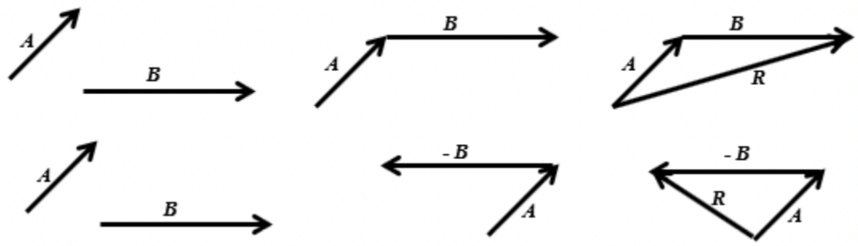
Parallelogram



Tip to Tail

A + B

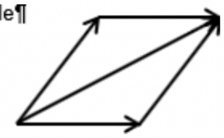
A – B



It is readily apparent that both methods arrive at the exact same solution since either method is essentially a parallelogram. It is useful to understand both systems. In some problems one method is advantageous, while inother problems the alternative method is superior.

1. Draw the resultant vector using the parallelogram method of vector addition.

Example¶



d.

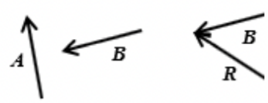
a. → ¶

e.

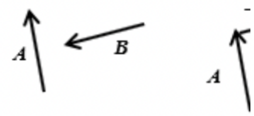
b. → ¶

2. Draw the resultant vector using the tip to tail method of vector addition. Label the resultant as vector ***R***

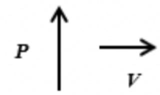
Example 1: *A* + *B*



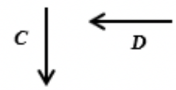
Example 2: *A* – *B*



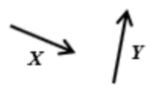
c. *P* + *V*



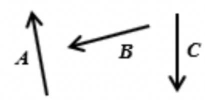
d. C – D



a. *X* + *Y*

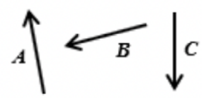


e. A + B + C

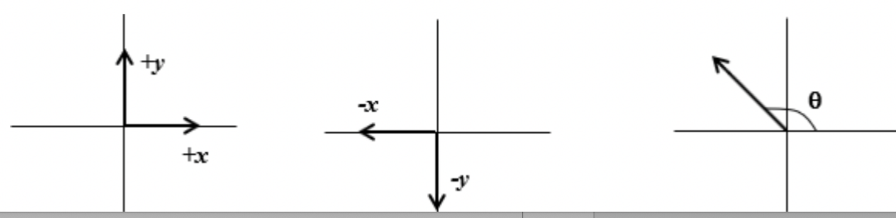


b. *T* – *S*

f. A – B – C



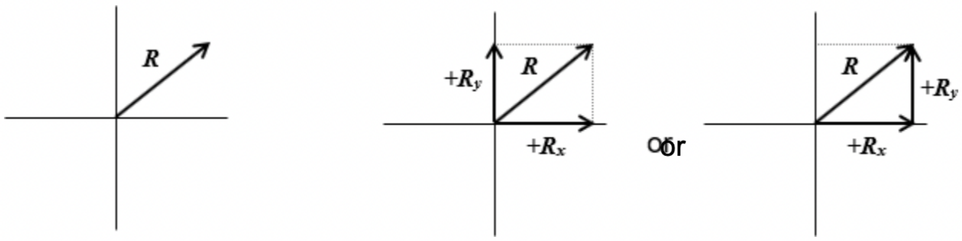
Direction: What does positive or negative direction mean? How is it referenced? The answer is the coordinate axis system. **In physics a coordinate axis system is used to give a problem a frame of reference.** Positive direction is a vector moving in the positive ***x*** or positive ***y*** direction, while a negative vector moves in the negative ***x***or negative ***y*** direction (This also applies to the ***z*** direction).



Component Vectors

A resultant vector is a vector resulting from the sum of two or more other vectors. Mathematically the resultant has the same magnitude and direction as the total of the vectors that compose the resultant. Could a vector be described by two or more other vectors? Would they have the same total result?

This is the reverse of finding the resultant. You are given the resultant and must find the component

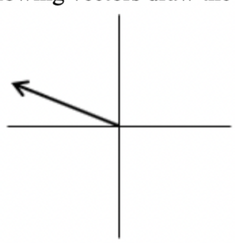


vectorson the coordinate axis that describe the resultant.

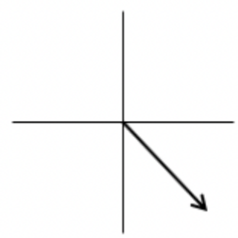
Any vector can be described by an ***x*** axis vector and a ***y*** axis vector which summed together mean the exactsame thing. The advantage is you can then use plus and minus signs for direction instead of the angle.

3. For the following vectors draw the component vectors along the ***x*** and ***y*** axis.

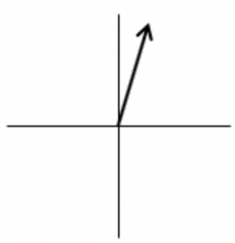
a.



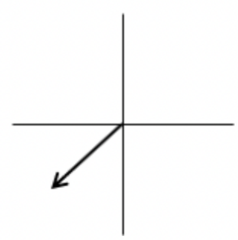
c.



b.

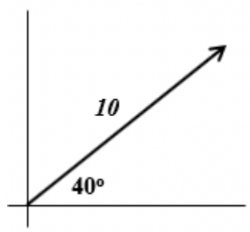


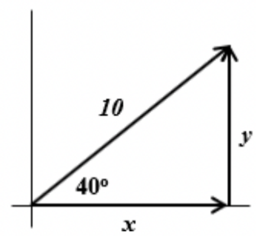
d.



Trigonometry and Vectors

Given a vector, you can now draw the ***x*** and ***y*** component vectors. The sum of vectors ***x*** and ***y*** describe the vector exactly. But, how do you mathematically find the length of the component vectors? Use trigonometry.





$$\cos\theta = \frac{adj}{hyp} \qquad \sin\theta = \frac{opp}{hyp}$$

$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$x = hyp\cos\theta \qquad y = hyp\sin\theta$$

$$x = 10\cos 40^\circ \qquad y = 10\sin 40^\circ$$

$$x = 7.66 \quad y = 6.43$$

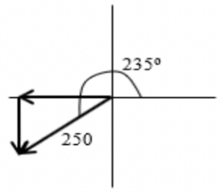
4. Solve the following problems. You will be converting from a polar vector, where direction is specified in ***<u>degrees measured counterclockwise from the positive x-axis</u>*** to component vectors along the ***x*** and ***y*** axis. Rememberthe plus and minus signs on you answers. They correspond with the quadrant the original vector is in.

   Hint: Draw the vector first to help you see the quadrant. Anticipate the sign on the ***x*** and ***y*** vectors. Do notbother to change the angle to less than 90º. Using the number given will result in the correct + and – signs.

   The first number will be the magnitude (length of the vector) and the second the degrees from east.

   Your calculator must be in degree mode.

Example: 250 at 235°



$x = hyp \cos\theta$

$x = 250 \cos 235^\circ$

$x = -143$

$y = hyp \sin\theta$

$y = 250 \sin 235^\circ$

$y = -205$

c. 0.00556 at 60°

a. 89 at 150°

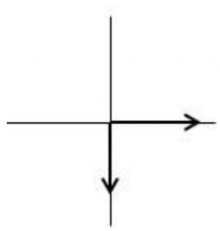
d. 7.5×10^4 at 180°

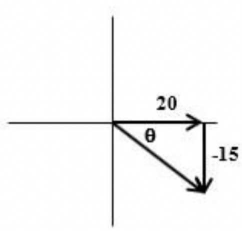
b. 6.50 at 345°

e. 12 at 265°

5. Given two component vectors solve for the resultant vector. This is the opposite of number 11 above. UsePythagorean Theorem to find the hypotenuse, then use inverse (arc) tangent to solve for the angle.

Example: ***x*** = 20, ***y*** = -15





$R^2 = x^2 + y^2$

$R = \sqrt{x^2 + y^2}$

$R = \sqrt{20^2 + 15^2}$

$R = 25$

$\theta = \tan^{-1}\left(\frac{-15}{20}\right) = -36.9^\circ$

$\tan\theta = \frac{opp}{adj}$

$\theta = \tan^{-1}\left(\frac{opp}{adj}\right)$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$360^\circ - 36.9^\circ = 323.1^\circ$

a. $x = 600$, $y = 400$

d. $x = 0.0065$, $y = -0.0090$

b. $x = -0.75$, $y = -1.25$

e. $x = 20{,}000$, $y = 14{,}000$

c. $x = -32$, $y = 16$

f. $x = 325$, $y = 998$

Estimated time to complete this section: 2 ½ hours.

If you have no calculus background whatsoever you may want to do some research (again YouTube videos) on the following topics: Derivatives and Power Rule, Chainrule and product rule, Other common derivatives, Derivatives and max/min, Definite integrals, Indefinite integrals. There are literally an infinite number of videos on these topics so I won't recommend any particular set. Find ones that you understand. You might want to watch them twice and take notes. Do your best. If you are in Calc AB this coming fall you might not be able to do all of this, but you should at least try.

Find the derivative of each of the following functions and simplify.

1. $f(x) = 4x^2 - 6$

2. $f(x) = 5x^3 - 3x$

3. $f(x) = 4x^3 - 3x^2 + 2x - \pi$

Find the antiderivative (integral) of each of the following.

1. $\int (3x^2 + 2x + 1)\,dx$

2. $\int_0^1 (x^5 + 5x + 8)\,dx$